

NAME: _____

This exam should have 4 pages; please check that it does.

(1) (20 points) Find the derivative, $f'(x)$, of the following functions:

(a) $f(x) = 5x^2 + \frac{1}{\sqrt{x}} + 1$

(b) $f(x) = (x^3 + 3x^2 + 1)(x^2 + 4)$

(c) $f(x) = \frac{x^2 + 1}{4x^2 + 2}$

(d) $f(x) = \frac{(\sqrt{x} + x)(x^2 + 5)}{2x + 1}$

(2) (10 points) Find the values of the following limits (part (b) is on the next page):

(a) $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 3x + 2}$

(b) $\lim_{x \rightarrow 2^-} f(x)$ where $f(x) = \begin{cases} 2x - 1, & x < 2 \\ x + 2, & x \geq 2 \end{cases}$

(3) (**points**)

(a) Find the equation of the **tangent line** to the graph of the curve $y = x^3 - 2x^2 + 1$ at the point $(2, 1)$.

(b) The graph shows the curve $y = f(x)$. Sketch the graphs of the tangent lines at the points $(a, f(a))$ and $(b, f(b))$.

(4) Find the derivative of the following functions:

(a) $y = \sqrt{4x + 1}$

(b) $y = x(x^2 + 1)^{\frac{1}{3}}$

- (5) Find a constant number a such that the following function is continuous on the entire real line:

$$f(x) = \begin{cases} x^3, & x \leq 2 \\ ax^2, & x > 2 \end{cases}$$

- (6) NOTE: The two parts of this question are about two **different** companies. Do NOT use the results of one part in the other part.

(a) A company sells MP3 players. When the price is \$95, they sell 100,000. If they drop the price to \$85, they sell 150,000. Assuming the **demand function** is linear, give a formula for the demand function.

(b) A company sells pizza. The **demand function** is $p = 8.5 - 0.001x$. If they sell 2,000 pizzas, what is their **revenue**?

- (7) This question asks you to complete the picture below:

- (a) Fill in the missing coordinates of the points.
(b) Draw the **secant line** between the two points.
(c) Filling the **rise** and the **run** as indicated.

(8) Use limits to find the derivative of $f(x) = 2x^2 + x$.

(9) When a certain company sells x items, its revenues $R(x)$ are given by the formula

$$R(x) = 25x - 0.001x^2$$

(a) What are the **marginal revenues** when the company sells 5,000 items?

(b) The company needed to invest \$10,000 to start production, but after that, each item costs \$5 to make. What is the **cost function**, $C(x)$, for this product?

(c) What is the **profit function** for this product?