NAME:_____

This exam should have 4 pages; please check that it does.

(1) **(20 points)** Find the derivative, $f'(x)$, of the following functions:

(a) $f(x) = 5x^2 + \dfrac{1}{\sqrt{x}} + 1$ $\qquad \frac{1}{\sqrt{x}} = x^{-1/2}$

$10x - \frac{1}{2}x^{-3/2}$

(b) $f(x) = (x^3 + 3x^2 + 1)(x^2 + 4)$ product rule

$(3x^2 + 6x)(x^2+4) + (x^3+3x^2+1)\cdot 2x$

(c) $f(x) = \dfrac{x^2+1}{4x^2+2}$ quotient rule

$$\frac{2x(4x^2+2) - (x^2+1)8x}{(4x^2+2)^2}$$

(d) $f(x) = \dfrac{(\sqrt{x}+x)(x^2+5)}{2x+1}$ product and quotient

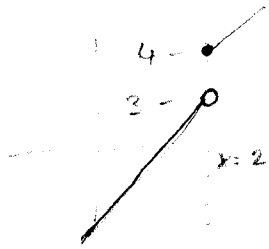
$$\frac{\left(\left(\frac{1}{2}x^{-\frac{1}{2}}+1\right)(x^2+5) + (\sqrt{x}+x)2x\right)(2x+1) - (\sqrt{x}+x)(x^2+5)\cdot 2}{(2x+1)^2}$$

(2) **(10 points)** Find the values of the following limits (part (b) is on the next page):

(a) $\lim\limits_{x\to 1} \dfrac{x^2+x-2}{x^2-3x+2}$

$$= \lim_{x\to 1} \frac{(x-1)(x+2)}{(x-1)(x-2)} = \frac{3}{-1} = -3$$

(b) $\lim_{x\to 2^-} f(x)$ where $f(x) = \begin{cases} 2x-1, & x<2 \\ x+2, & x\geq 2 \end{cases}$



$\lim_{x\to 2^-} f(x) = 3$

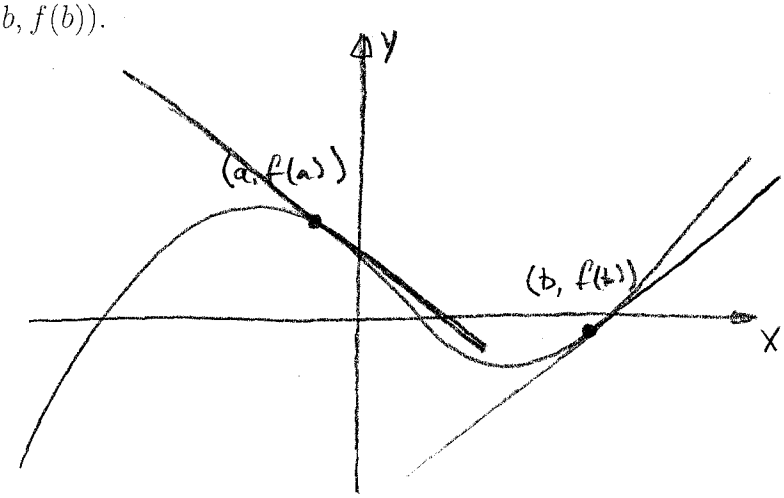
(3) (**points**)

(a) Find the equation of the **tangent line** to the graph of the curve $y = x^3 - 2x^2 + 1$ at the point $(2, 1)$.

$\frac{dy}{dx} = 3x^2 - 4x$; when $x = 2$ slope is $3\cdot 2^2 - 4\cdot 2 = 4$

tangent line: $y - 1 = 4(x-2)$

(b) The graph shows the curve $y = f(x)$. Sketch the graphs of the tangent lines at the points $(a, f(a))$ and $(b, f(b))$.



(4) Find the derivative of the following functions:

(a) $y = \sqrt{4x+1}$

$= u^{\frac{1}{2}}$

$u = 4x+1$

$\frac{dy}{dx} = \frac{1}{2}u^{-\frac{1}{2}} \cdot 4 = \frac{1}{2}(4x+1)^{-\frac{1}{2}} \cdot 4$

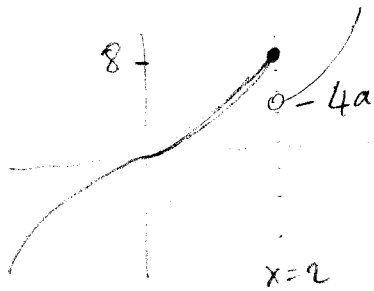
(b) $y = x(x^2+1)^{\frac{1}{3}}$ product and chain $u^{1/3}$; $u = x^2+1$

$\frac{dy}{dx} = 1(x^2+1)^{\frac{1}{3}} + x\left(\frac{1}{3}(x^2+1)^{-2/3} \cdot 2x\right)$

$\frac{1}{2}u^{-2/3}$ $\frac{du}{dx}$

(5) Find a constant number a such that the following function is continuous on the entire real line:

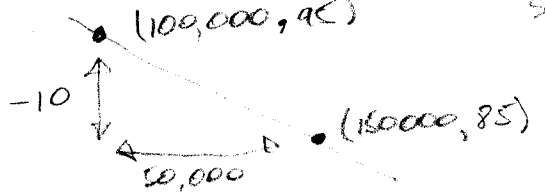
$$f(x) = \begin{cases} x^3, & x \le 2 \\ ax^2, & x > 2 \end{cases}$$



need $4a = 8$ so $a = 2$

(6) NOTE: The two parts of this question are about two **different** companies. Do NOT use the results of one part in the other part.

(a) A company sells MP3 players. When the price is \$95, they sell 100,000. If they drop the price to \$85, they sell 150,000. Assuming the **demand function** is linear, give a formula for the demand function.



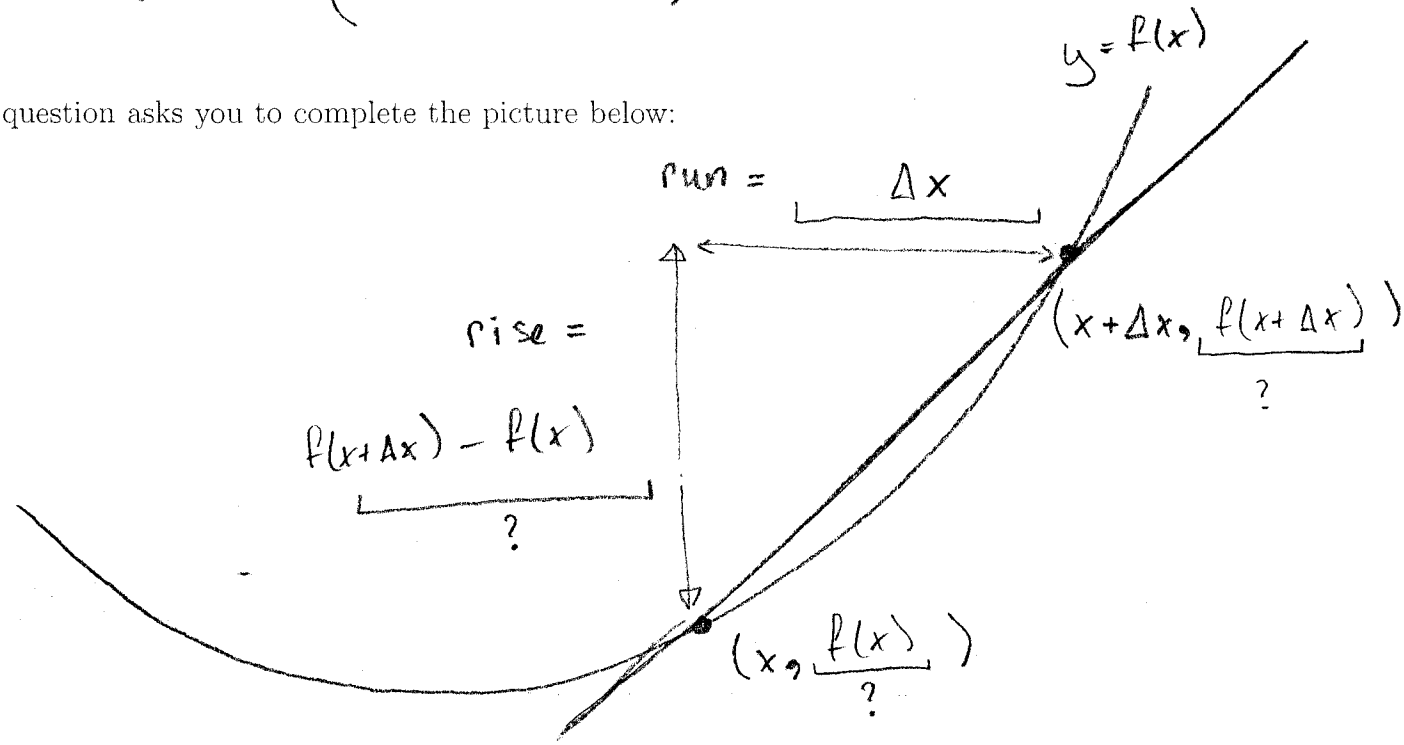
slope $= \frac{-10}{50000}$ $\quad p = f(x)$

$$y - 85 = \frac{-10}{50000}(x - 150000)$$

(b) A company sells pizza. The **demand function** is $p = 8.5 - 0.001x$. If they sell 2,000 pizzas, what is their **revenue**?

$$R(x) = x(8.5 - 0.001x)$$

(7) This question asks you to complete the picture below:



(a) Fill in the missing coordinates of the points.
(b) Draw the **secant line** between the two points.
(c) Filling the **rise** and the **run** as indicated.

(8) Use limits to find the derivative of $f(x) = 2x^2 + x$.

$$\lim_{\Delta x \to 0} \frac{\left(2(x+\Delta x)^2 + (x+\Delta x)\right) - (2x^2 + x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x^2 + 4x\Delta x + 2\Delta x^2 + x + \Delta x - 2x^2 - x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4x\Delta x + 2\Delta x^2 + \Delta x}{\Delta x} = \lim_{\Delta x \to 0} 4x + 2\Delta x + 1 = 4x + 1$$

(9) When a certain company sells x items, its revenues $R(x)$ are given by the formula

$$R(x) = 25x - 0.001x^2$$

(a) What are the **marginal revenues** when the company sells 5,000 items?

$$R'(x) = 25 - 0.001x$$

$$R'(5000) = 25 - 0.001 \times 5000 = 25 - 5 = 20$$

(b) The company needed to invest \$10,000 to start production, but after that, each item costs \$5 to make. What is the **cost function**, $C(x)$, for this product?

$$C(x) = 10000 + 5x$$

(c) What is the **profit function** for this product?

$$P(x) = R(x) - C(x)$$

$$= 25x - 0.001x^2 - 10000 - 5x$$

$$R(x) \qquad - C(x)$$