November 5, 2010 **MATH 104 (John Orr's Section)** **EXAM III**

NAME:_____

This exam should have 5 pages; please check that it does.

Question:	1	2	3	4	5	6	Total
Points:	16	26	10	12	16	20	100
Score:							

1. The population P in thousands of Houston, Texas from 1980 through 2005 can be modeled by

$$P = 1576e^{0.01t},$$

where $t = 0$ corresponds to 1980.

(a) (8 points) According to the model, what was the population of Houston in 2005?

(b) (8 points) According to the model, in what year will Houston have a population of 2,500,000? (Hint: Remember P is the population in *thousands*.)

2. The **demand function** for a product is $p = 450 - 0.25x$.

   (a) (5 points) What price should you set, to get sales levels of $1,000$ units?

   (b) (5 points) Find the **revenue function**, R.

   (c) (4 points) What **sales level**, x, maximizes R?

   (d) (4 points) What **price**, p, maximizes R?

   (e) (4 points) Find the **elasticity** η for this product.

   (f) (4 points) Use your answer to confirm that the product has **unit elasticity** when the revenue is maximized.

3. (a) (5 points) The profit function for a product is $P = -0.5x^3 + 2500x - 6000$. Find the **differential** dP.

   (b) (5 points) A business analyzes its revenues R and calculates the differential dR to be

   $$dR = (30 - 0.3x)dx$$

   If the current sales level is $x = 50$, what change in revenues do you estimate for an increase in sales of $dx = 1$?

4. After t years, the remaining mass y (in grams) of 16 grams of a radioactive element is given by

   $$y = 16\left(\frac{1}{2}\right)^{t/25}$$

   (a) (6 points) What is the **half-life** of the element?

   (b) (6 points) How much remains after 100 years?

5. (a) (8 points) How much money will you have if you invest \$10,000 at 5% interest **compounded quarterly** for 20 years?

   (b) (8 points) How much money will you have if you invest the \$10,000 at 5% interest **compounded continuously** for 20 years?

6. Find $\dfrac{dy}{dx}$ for the following:

   (a) (5 points) $y = e^{4x^2}$

   (b) (5 points) $y = (x^2 + 2)e^{-3x}$

(c) (5 points) $y = x^2 \ln(x)$

(d) (5 points) $y = \dfrac{\ln(x)}{x^2 + 1}$